Two Clause Learning Approaches for Disjunctive Scheduling

Mohamed Siala, Christian Artigues, and Emmanuel Hebrard

Context

Disjunctive Scheduling

A family of scheduling problems having in common the `Unary Resource Constraint`.

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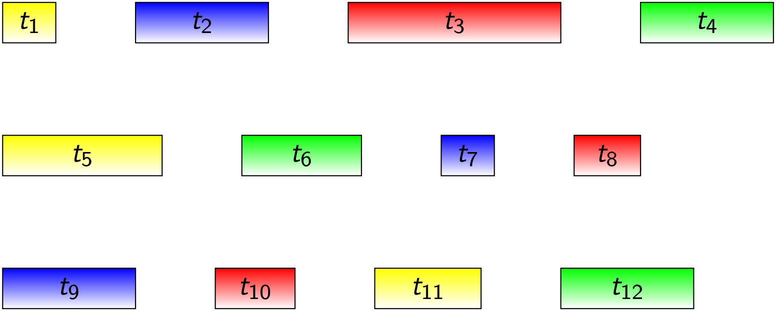

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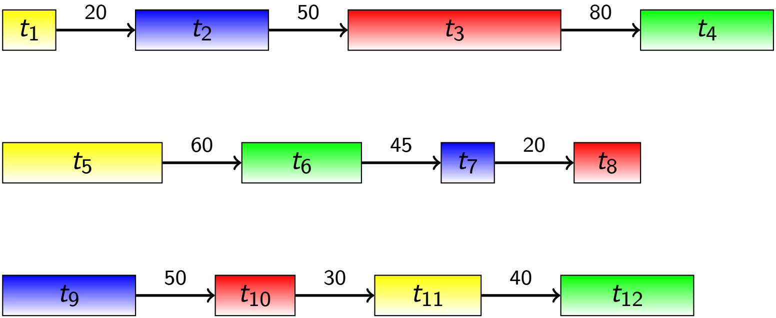

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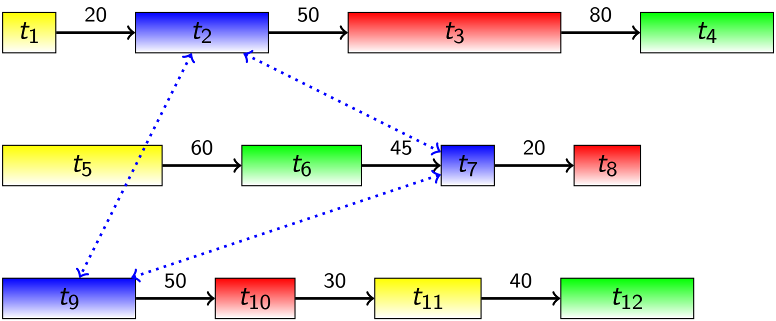

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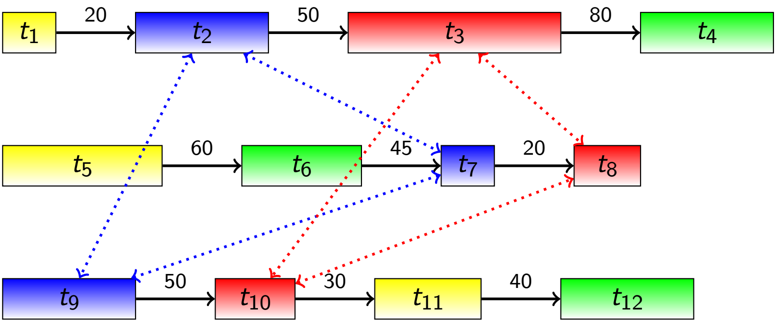

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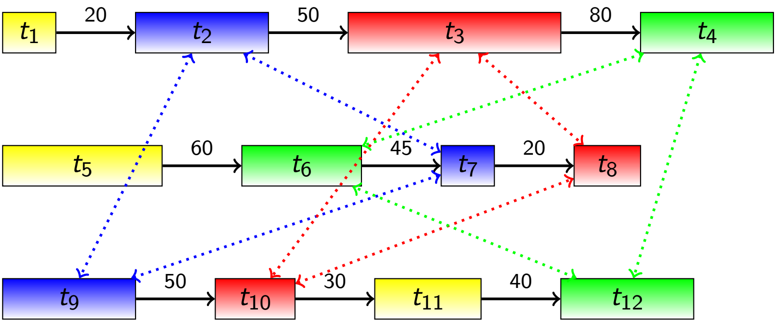

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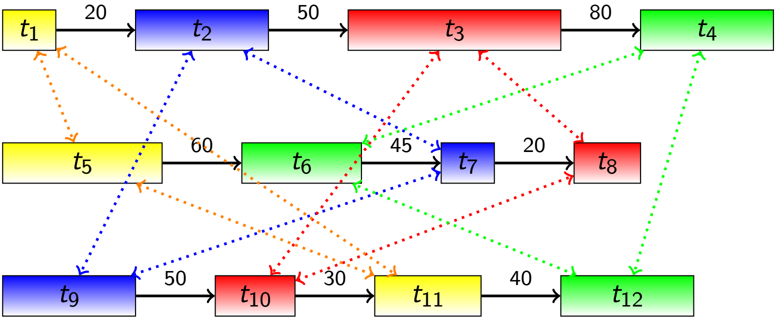

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Scheduling in CP

- Tradition
  - Tailored propagation algorithms (such as `Edge-Finding` [Carlier and Pinson, 1989]
  - Tailored search strategies (such as `Texture` [Sadeh, 1991]).

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Scheduling in CP

- Tradition
  - Tailored propagation algorithms (such as `Edge-Finding` [Carlier and Pinson, 1989]
  - Tailored search strategies (such as `Texture` [Sadeh, 1991]).
- New trend: Focus on what can be learnt during search
  - Lazy Clause Generation for `RCPSP` [Schutt et al., 2013].
  - Weight-based heuristic learning on disjunctive scheduling [Grimes and Hebrard, 2015].

Example

$$x_1 + x_7 \geq 4 \wedge$$
$$x_2 + x_{10} \geq 11 \wedge$$
$$x_3 + x_9 = 16 \wedge$$
$$x_5 \geq x_8 + x_9 \wedge$$
$$b \leftrightarrow (x_9 - x_4 = 14) \wedge$$
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b	0	1
x_1	1	30
x_2	9	30
x_3	0	3
x_4	0	30
x_5	24	45
x_6	5	10
x_7	2	10
x_8	9	30
x_9	13	16
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x_{10}	0	3
x_{11}	15	25

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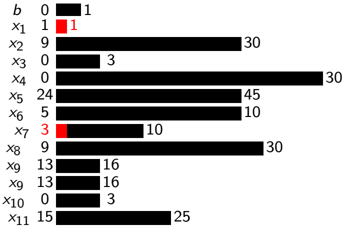

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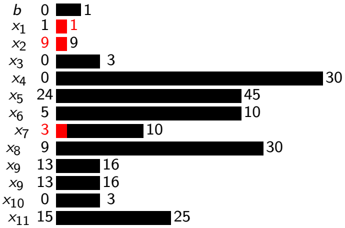

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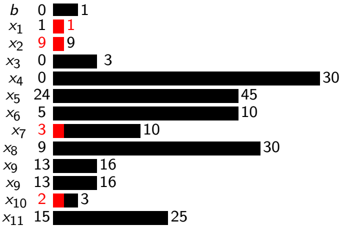

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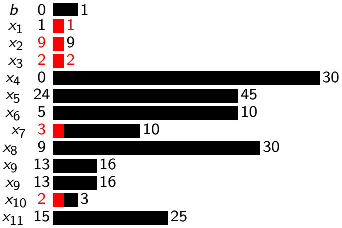

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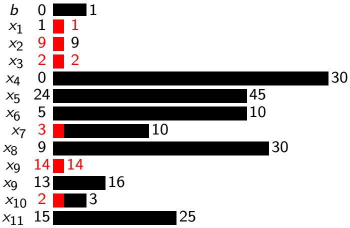

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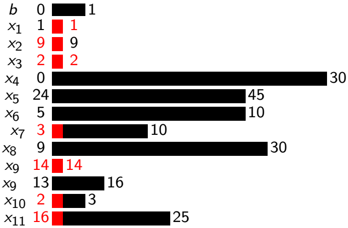

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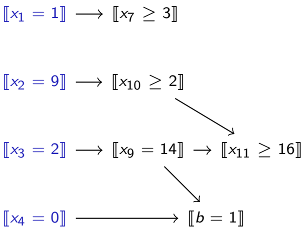

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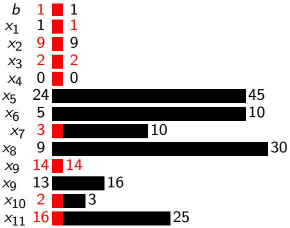

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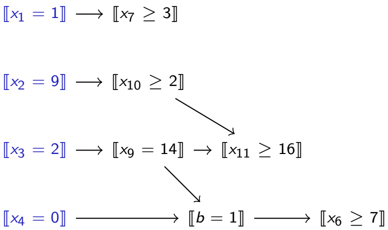

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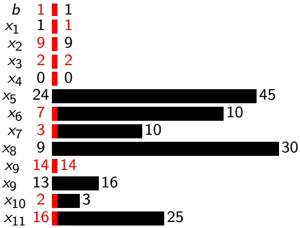

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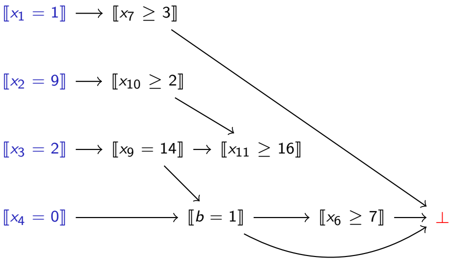

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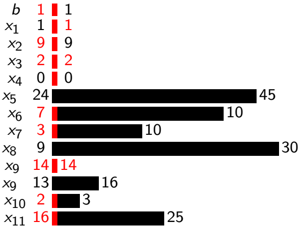

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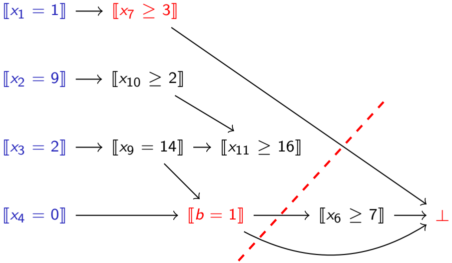

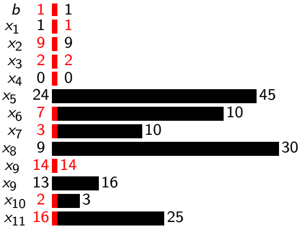

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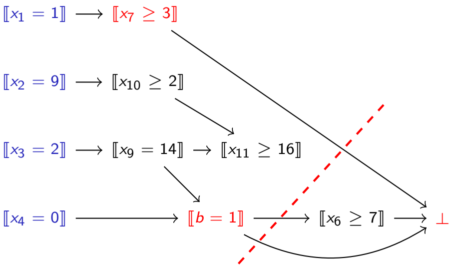

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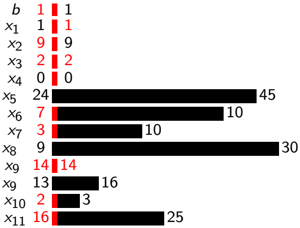

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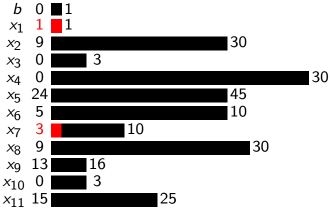

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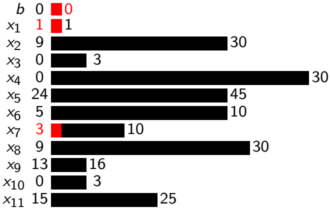

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- Backtrack to level 1
- Propagate the learnt clause
- Continue exploration

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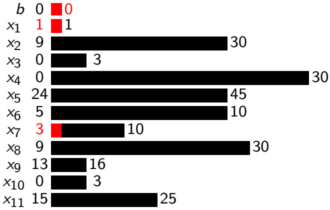

Learning in CP

- Hybrid CP/SAT
- Conflict Driven Clause Learning (CDCL) [Moskewicz et al., 2001]
- Based on the notion of explanation
- Forward/Backward explanations
- Domain atoms can be generated Eagerly/Lazily

Our contributions

- Alternative lazy (atom) generation approach
- Novel conflict analysis scheme tailored to disjunctive scheduling

Modelling [Grimes and Hebrard, 2015]

Unary Resource Constraint

- $O(n^2)$ Boolean variables δ_{kij} $(i < j \in [1, n])$ per machine M_k.
- Decomposition using the following DISJUNCTIVE constraints:

$$\delta_{kij} = \begin{cases} 0 & \Leftrightarrow \quad t_{ik} + p_{ik} \leq t_{jk} \\ 1 & \Leftrightarrow \quad t_{jk} + p_{jk} \leq t_{ik} \end{cases} \tag{1}$$

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- Variable ordering: wdeg, VSIDS.
- Value ordering: Solution guided [Beck, 2007].
- Greedy heuristic, dichotomic search, branch and bound

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Domain Encoding: standard approach

1. Generate domain atoms: $a \leftrightarrow [\![x = d]\!]$, $b \leftrightarrow [\![x \leq d]\!]$
2. Generate domain clauses: $\neg[\![x \leq d]\!] \vee [\![x \leq d+1]\!]$, $\neg[\![x = d]\!] \vee [\![x \leq d]\!]$, etc

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Lazy Atom Generation

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2. There is a redundancy issue
3. Suppose that $[\![x \leq 2]\!]$, $[\![x \leq 4]\!]$ are already generated with $\neg[\![x \leq 2]\!] \vee [\![x \leq 4]\!]$ and we will generate $[\![x \leq 3]\!]$.
4. Add $\neg[\![x \leq 2]\!] \vee [\![x \leq 3]\!]$, $\neg[\![x \leq 3]\!] \vee [\![x \leq 4]\!]$.
5. $\neg[\![x \leq 2]\!] \vee [\![x \leq 4]\!]$ is useless (redundant)
6. For a domain of size k, $k - 2$ redundant clauses.

Avoiding redundancy via DomainFaithfulness

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Key Idea

- Use a single constraint responsible for the consistency of the domain.
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$\textsc{DomainFaithfulness}(x, [b_1 \ldots b_n], [v_1, \ldots, v_n]) : \forall i, b_i \leftrightarrow x \leq v_i$

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Arc consistency

Can be enforced in constant amortized time complexity ($O(1)$) down a branch of the search tree

DISJUNCTIVE-based Learning

DISJUNCTIVE-based Learning

- Branch on the reified Boolean variables
- $\rightarrow$ There exists an explanation for every bound literal $[\![x \leq u]\!]$

Disjunctive-based Learning

- Branch on the reified Boolean variables
- $\rightarrow$ There exists an explanation for every bound literal $[\![x \leq u]\!]$

Disjunctive-based Learning

Two phases:

1. 1-UIP cut
2. Apply resolution for every bound literal until having a nogood with only reified Boolean variables

DISJUNCTIVE-based Learning

Example

- 1-UIP nogood: $a \wedge \neg b \wedge [\![x \leq 7]\!] \wedge [\![y \leq 9]\!] \Rightarrow \bot$

DISJUNCTIVE-based Learning

Example

- 1-UIP nogood: $a \wedge \neg b \wedge [\![x \leq 7]\!] \wedge [\![y \leq 9]\!] \Rightarrow \bot$
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- $a \wedge [\![x \geq 0]\!] \Rightarrow [\![z \leq 13]\!]$

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Disjunctive-based Learning

Example

- 1-UIP nogood: $a \wedge \neg b \wedge [\![x \leq 7]\!] \wedge [\![y \leq 9]\!] \Rightarrow \perp$
- $c \wedge [\![z \leq 13]\!] \Rightarrow [\![x \leq 7]\!]$
- Resolution $a \wedge \neg b \wedge c \wedge [\![z \leq 13]\!] \wedge [\![y \leq 9]\!] \Rightarrow \perp$
- $[\![x \geq 4]\!] \Rightarrow [\![y \leq 9]\!]$
- Resolution $a \wedge \neg b \wedge c \wedge [\![z \leq 13]\!] \wedge [\![x \geq 4]\!] \Rightarrow \perp$
- $a \wedge [\![x \geq 0]\!] \Rightarrow [\![z \leq 13]\!]$
- Resolution $a \wedge \neg b \wedge c \wedge a \wedge [\![x \geq 0]\!] \wedge [\![y \geq 4]\!] \Rightarrow \perp$
- Nogood Reduction $a \wedge \neg b \wedge c \wedge [\![y \geq 4]\!] \Rightarrow \perp$

DISJUNCTIVE-based Learning

Example

- 1-UIP nogood: $a \wedge \neg b \wedge [\![x \leq 7]\!] \wedge [\![y \leq 9]\!] \Rightarrow \bot$
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- Resolution $a \wedge \neg b \wedge c \wedge [\![z \leq 13]\!] \wedge [\![y \leq 9]\!] \Rightarrow \bot$
- $[\![x \geq 4]\!] \Rightarrow [\![y \leq 9]\!]$
- Resolution $a \wedge \neg b \wedge c \wedge [\![z \leq 13]\!] \wedge [\![x \geq 4]\!] \Rightarrow \bot$
- $a \wedge [\![x \geq 0]\!] \Rightarrow [\![z \leq 13]\!]$
- Resolution $a \wedge \neg b \wedge c \wedge a \wedge [\![x \geq 0]\!] \wedge [\![y \geq 4]\!] \Rightarrow \bot$
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- $\neg b \wedge [\![z \geq 0]\!] \Rightarrow [\![y \geq 4]\!]$
- Resolution $a \wedge \neg b \wedge c \Rightarrow \bot$

- ⊕ No domain encoding
- ⊕ Scheduling horizon does not manner in size
- ⊖ Language of literals is restricted compared to standard approaches

Experimental results

Protocol

- **Mistral-Hybrid:** backward explanations, semantic reductions, lazy generation, DISJUNCTIVE-based learning
- **http://siala.github.io/jssp/details.pdf**
- Lawrence and Taillard Job Shop benchmarks
- Global cutoff: 1h

Experimental results

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- **Mistral-Hybrid:** backward explanations, semantic reductions, lazy generation, DISJUNCTIVE-based learning
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- Global cutoff: 1h

How hard are Taillard instances?

- Proposed 2 decades ago
- State-of-the art method recently proposed in [Vilím et al., 2015]
  - IBM CP-Optimizer studio
  - 8h20min per instance
  - Parallelization
  - Start search with best known bounds as an additional information.

Experimental results: Summary

Experimental results: Summary

Mostly proven optimal

Instances	CP(*task*)			H(*vsids*, *disj*)			H(*vsids*, *lazy*)			H(*task*, *disj*)			H(*task*, *lazy*)		
	%O	T	nds/s	%O	T	nds/s	%O	T	nds/s	%O	T	nds/s	%O	T	nds/s
la-01-40	87	522	8750	**91.5**	437	6814	88	632	2332	90.50	434	5218	88.75	509	2694
tai-01-10	89	768	5875	**90**	517	4975	88	1060	1033	90	634	3572	84	1227	1013

Hard instances

Instances	CP(*task*)		H(*vsids*, *disj*)		H(*vsids*, *lazy*)		H(*task*, *disj*)		H(*task*, *lazy*)	
	PRD	nds/s	PRD	nds/s	PRD	nds/s	PRD	nds/s	PRD	nds/s
tai-11-20	1.8432	4908	**1.1564**	3583	1.3725	531	1.2741	2544	1.2824	489
tai-21-30	1.6131	3244	0.9150	2361	1.0841	438	0.9660	1694	**0.8745**	409
tai-31-40	5.4149	3501	4.0210	2623	**3.7350**	580	4.0536	1497	3.8844	510
tai-41-50	7.0439	2234	4.8362	1615	**4.6800**	436	4.9305	1003	5.0136	390
tai-51-60	3.0346	1688	3.2449	2726	3.7809	593	**1.1156**	1099	1.1675	575
tai-61-70	6.8598	1432	6.5890	2414	5.4264	578	3.9637	866	**3.6617**	533

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Instances	CP(*task*)			H(*vsids*, *disj*)			H(*vsids*, *lazy*)			H(*task*, *disj*)			H(*task*, *lazy*)		
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- Hybrid models outperform CP

Experimental results: Summary

Instances	CP(*task*)			H(*vsids*, *disj*)			H(*vsids*, *lazy*)			H(*task*, *disj*)			H(*task*, *lazy*)		
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- The impact of clause learning is more visible when the size of the instance grows

Experimental results: Summary

Instances	CP(*task*)			H(*vsids*, *disj*)			H(*vsids*, *lazy*)			H(*task*, *disj*)			H(*task*, *lazy*)		
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- DISJUNCTIVE-based learning outperforms the other models on medium sized instances

Experimental results: Summary

Instances	CP(*task*)			H(*vsids*, *disj*)			H(*vsids*, *lazy*)			H(*task*, *disj*)			H(*task*, *lazy*)		
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- wdeg is the best choice with the largest instances.

Experimental results: Summary

Instances	CP(*task*)			H(*vsids*, *disj*)			H(*vsids*, *lazy*)			H(*task*, *disj*)			H(*task*, *lazy*)		
Mostly proven optimal															
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- Surprisingly DISJUNCTIVE-based learns shorter clauses

Experimental results: lower bounds computation

Experimental results: lower bounds computation

Open instances from Taillard benchmark before [Vilím et al., 2015]

- 7 new bounds found with DISJUNCTIVE-based learning and VSIDS

tai13		tai21		tai23		tai25		tai26		tai29		tai30	
new	old	new	old	new	old	new	old	new	old	new	old	new	old
1305	1282	**1613**	1573	**1514**	1474	**1544**	1518	**1561**	1558	**1576**	1525	**1515**	1485

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tai13		tai21		tai23		tai25		tai26		tai29		tai30	
new	old	new	old	new	old	new	old	new	old	new	old	new	old
1305	1282	**1613**	1573	**1514**	1474	**1544**	1518	**1561**	1558	**1576**	1525	**1515**	1485
1342		**1642**		**1518**		**1558**		**1591**		**1573**		**1519**	

- 8h20min per instance
- Parallelization
- Start search with best known bounds as an additional information.

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- 8h20min per instance
- Parallelization
- Start search with best known bounds as an additional information.

Relaunch with 2h

- tai-29: 1583 (1573 in [Vilím et al., 2015])
- tai-30: 1528 (1519 in [Vilím et al., 2015])

Summary

- Alternative lazy (atom) generation approach avoiding a redundancy issue
- Novel conflict analysis mechanism
- Efficient in practice, specially for finding proofs

Future Research

- Applications to other scheduling problems?
- Learning with global constraints?
- Hand-crafted learning?

Thank you.

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