Centre for
Data Analytics

Insight

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Explanation-Based Weighted Degree

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DCU NUI Galway OÉ Gaillimh UCC University College Cork, Ireland Coláiste na hOllscoile Corcaigh UCD DUBLIN SFI Science Foundation Ireland

CPAIOR 2017

- International Conference on Integration of **Artificial Intelligence** and **Operations Research** Techniques in **Constraint Programming**

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- Diverse applications: Network Problems (interesting for the UTRC field service project?), Scheduling, Data-mining, Stochastic Optimization

MiniCP

- Pierre Schaus, Pascal Van Hentenryck, Laurent Michel
- 1500 lines of Java code
- Code: `https://bitbucket.org/pschaus/minicp`
- Slides: `http://tinyurl.com/y8n4knhx`
- Purposes: teaching CP, baseline for development projects, etc

Relaxation Methods for Constrained Matrix Factorization Problems: Solving the Phase Mapping Problem in Materials Discovery

- Phase mapping: Central problem in materials discovery
- How to infer the materials' crystal structure based on X-ray sample data.
- The problem: factorize A into the product of W and H such that $A = W \times H$.

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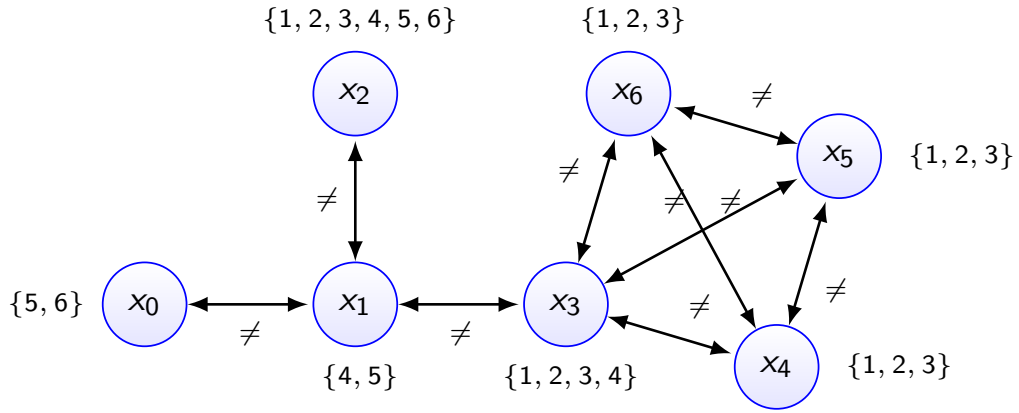
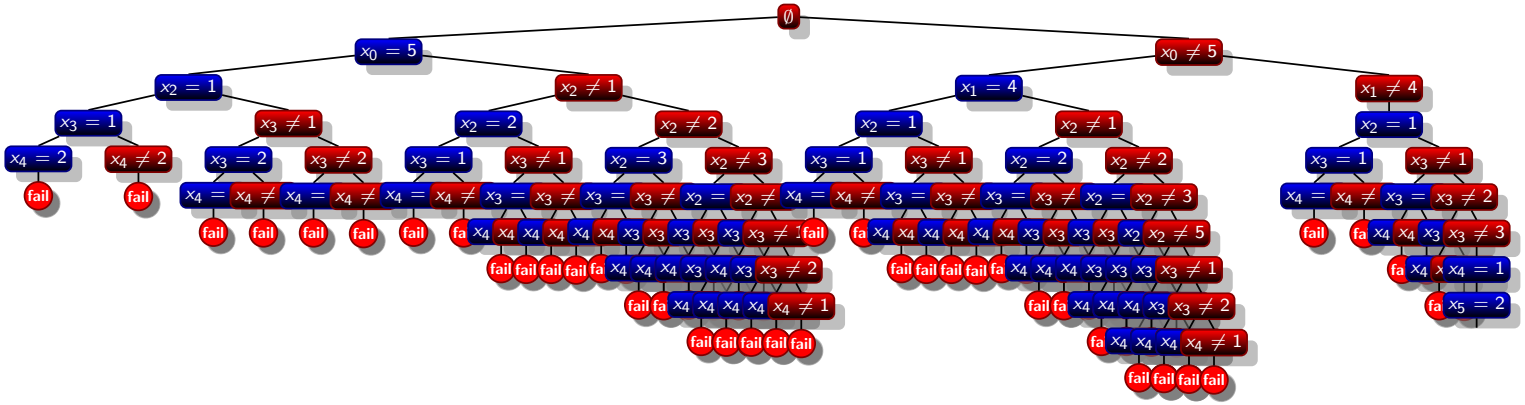
- Search: decisions to explore the search tree
- Search in CP = **variable ordering** + value ordering
- Standard or customized

Lexicographic





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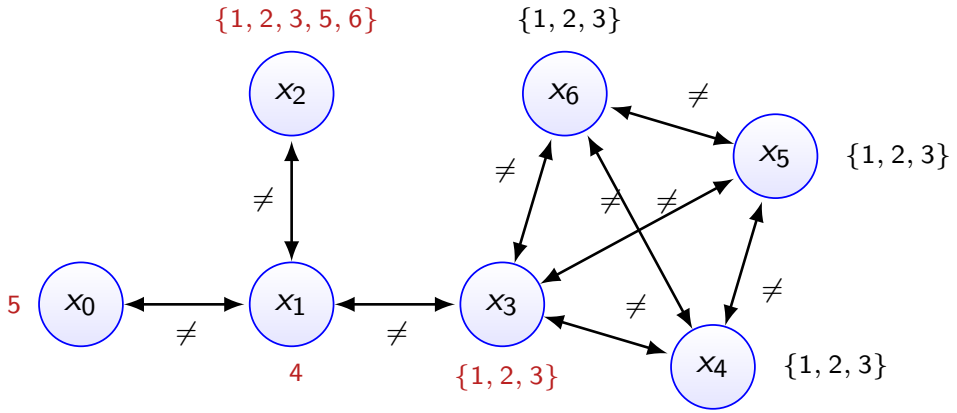

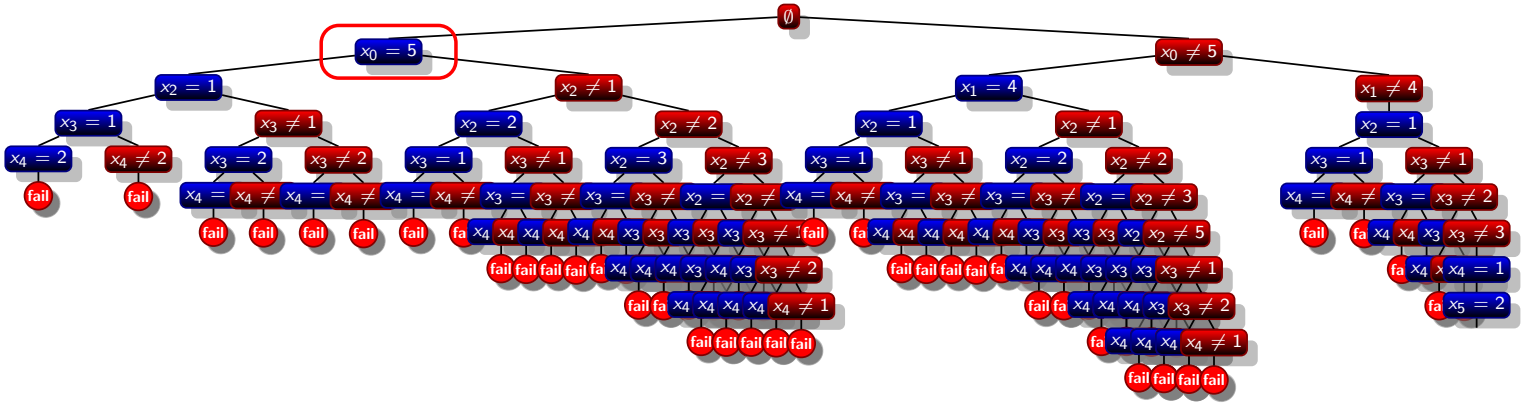

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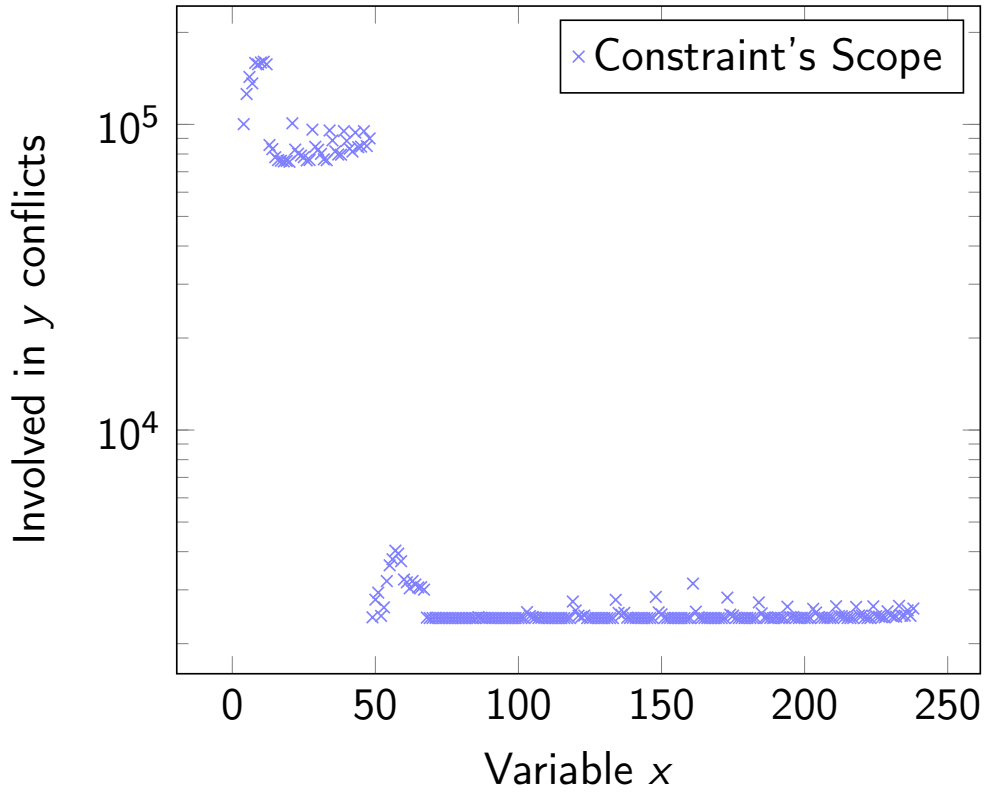
- **A big issue with global constraints as it does not discriminate between variables**

The Discrimination Issue

- An instance of ghoulomb.mzn (contains a lot of AllDifferent Constraints)



The "Flooding" Issue

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x_1, x_2 &\in [1, 2n-1] \\
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  - **wdeg now branches first on $x_3, \ldots, x_n$!**
- Weight distributed indiscriminately "masks" the degree

Explanation-based Weighted Degree

- **A simple solution:** Weight a subset of variables (**explanation**) responsible for failure instead of the whole constraint scope.

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- Example: $\sum_{i=1}^{n} x_i \leq k$: Variables whose domain is equal to $\{1\}$ are sole responsible for failure.

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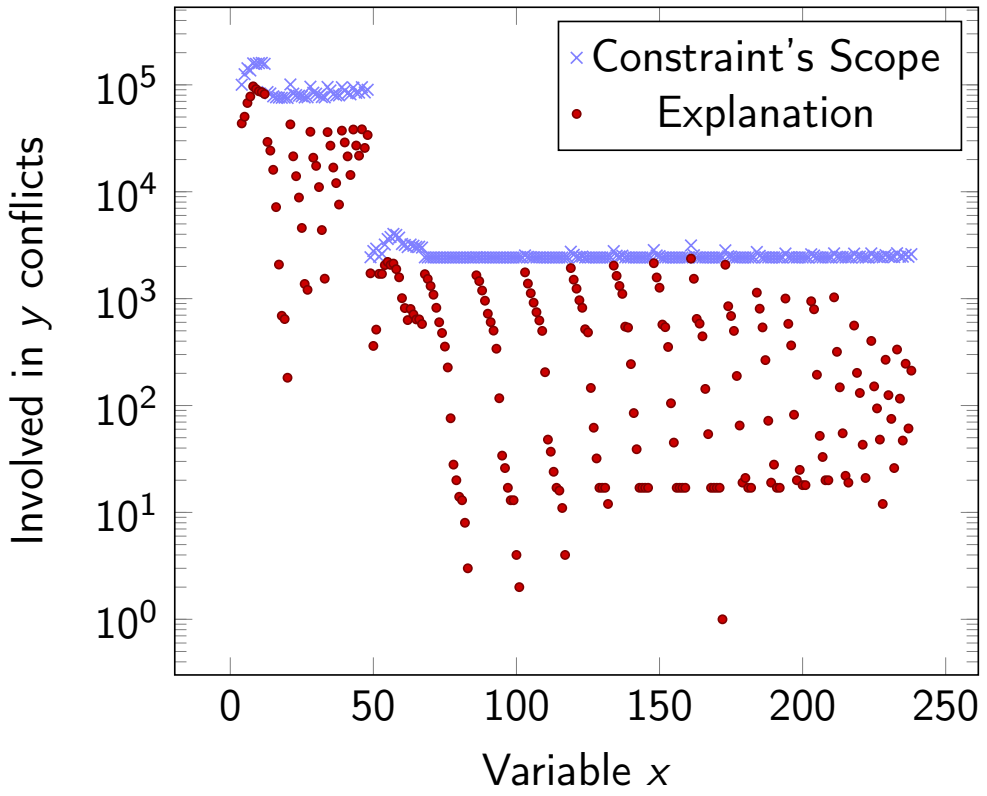

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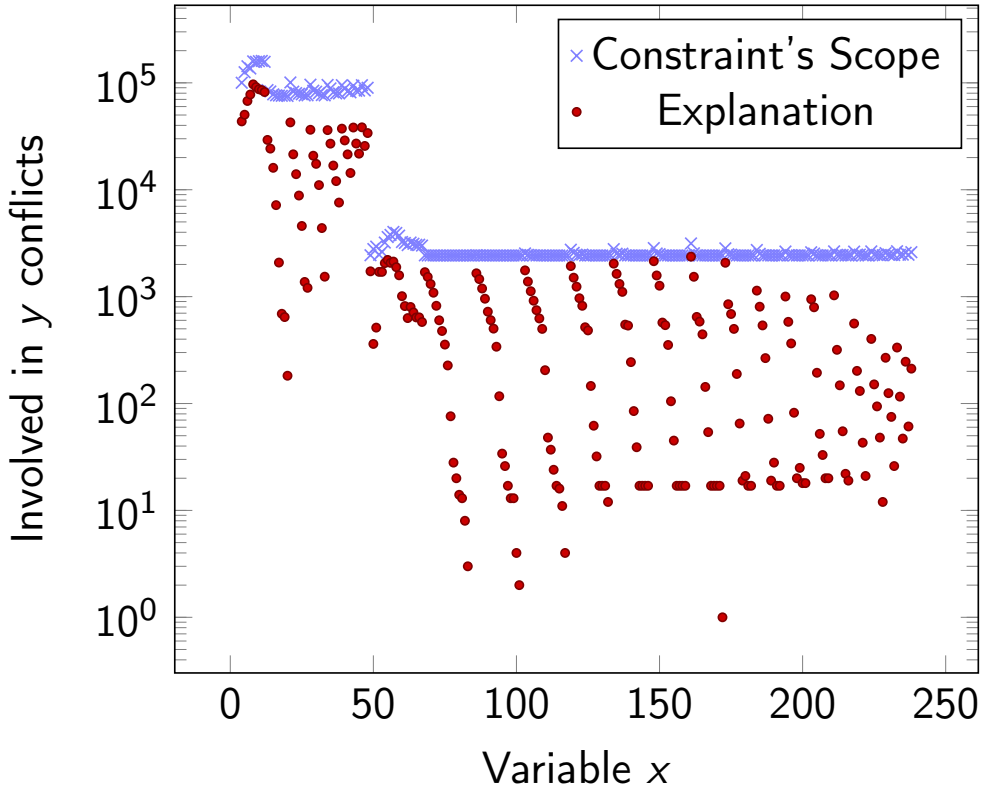

- *e-wdeg*: Optimal solution was proven 15s
- *wdeg*: 83% optimality gap after 20 minutes

Explanations for three Global Constraints

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AllDifferent

- Bound Consistency Propagator: Failure if there exists a Hall interval, i.e., a set of at least $b - a + 2$ variables whose domains are included in $\{a, \ldots, b\}$
- Explanation: the set of variables from the Hall interval.

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Element$(\langle x_1, \ldots, x_k \rangle, n, v) \Leftrightarrow x_n = v$,

- We use the conflict set $\{n, v\} \cup \{x_i \mid i \in \mathcal{D}(n)\}$

Explanations for three Global Constraints

$\sum_{i=1}^{k} a_i x_i \leq b$

- Failure if and only if the lower bound of the sum is strictly larger than b.
- Explanation: The set containing every variable x_i such that either a_i is positive and $\min(\mathcal{D}(x_i))$ is strictly larger than its initial value, or a_i is negative and $\max(\mathcal{D}(x_i))$ is strictly lower than its initial value.

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- Last Conflict (*LC*) [?] : Once both branches ($x = a$ and $x \neq a$) have failed, the variable x is always preferred (it needs a default heuristic).
- Conflict Ordering Search (*COS*) [?]: Every variable is stamped by the total number of failures it caused. The variable with the highest stamp is selected first (it needs a default heuristic)

Experimental Setup

- All Minizinc Challenge instances (2012 to 2015): 323 optimization problems; 76 decision problems
- Comparison with *wdeg*, *ABS*, *IBS*, *COS* and *LC* with *wdeg* and *e-wdeg* as default heuristic for the two latter.
- Lexicographic value ordering for every heuristic, except *IBS* and *ABS*.
- Randomization by choosing uniformly between the two best choices.
- Each configuration was given 5 randomized runs.
- 25 minutes as time cutoff

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- **In case you wonder why the cluster is busy: with some other parameters, the total CPU time to complete experiments is about 3 years and a half!**

Decision Problems

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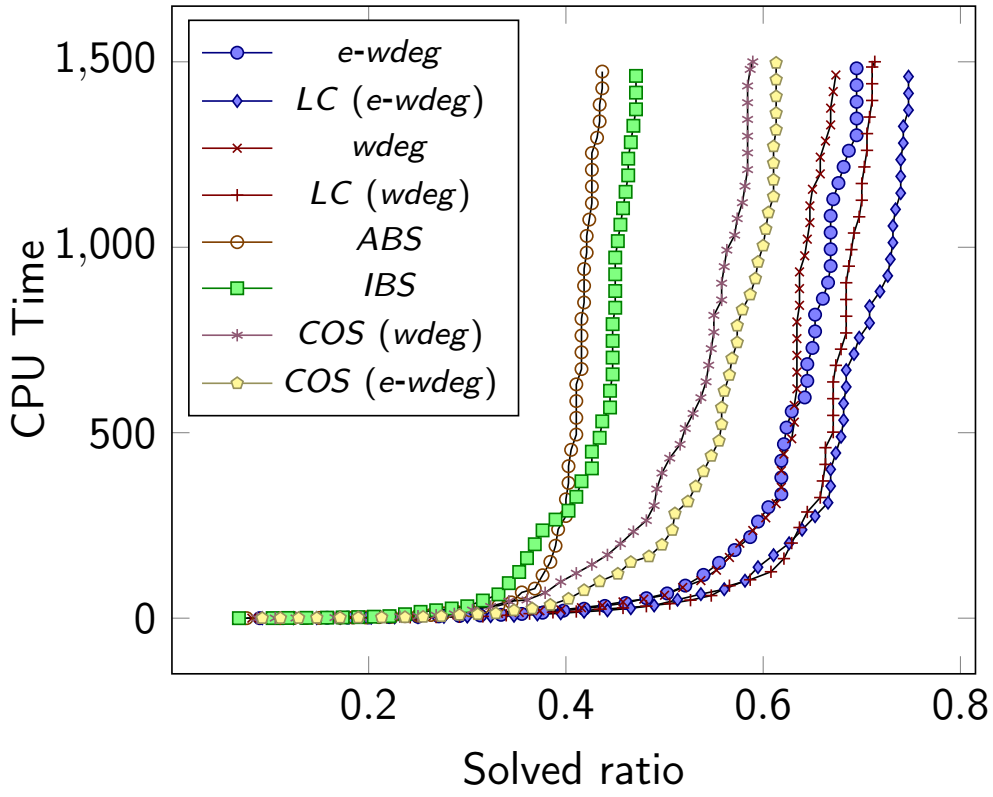

- *e-wdeg* better than *wdeg*
- Weighted degree heuristics are among the state of the art in CP

Optimisation Problems (Number of proofs)

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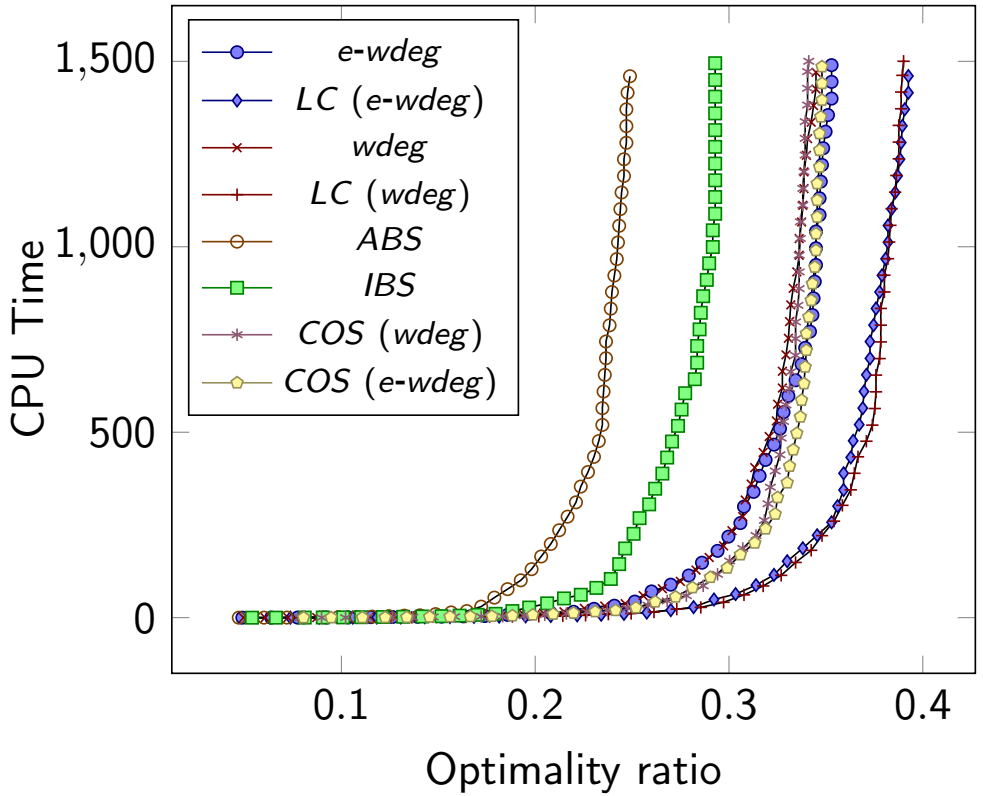

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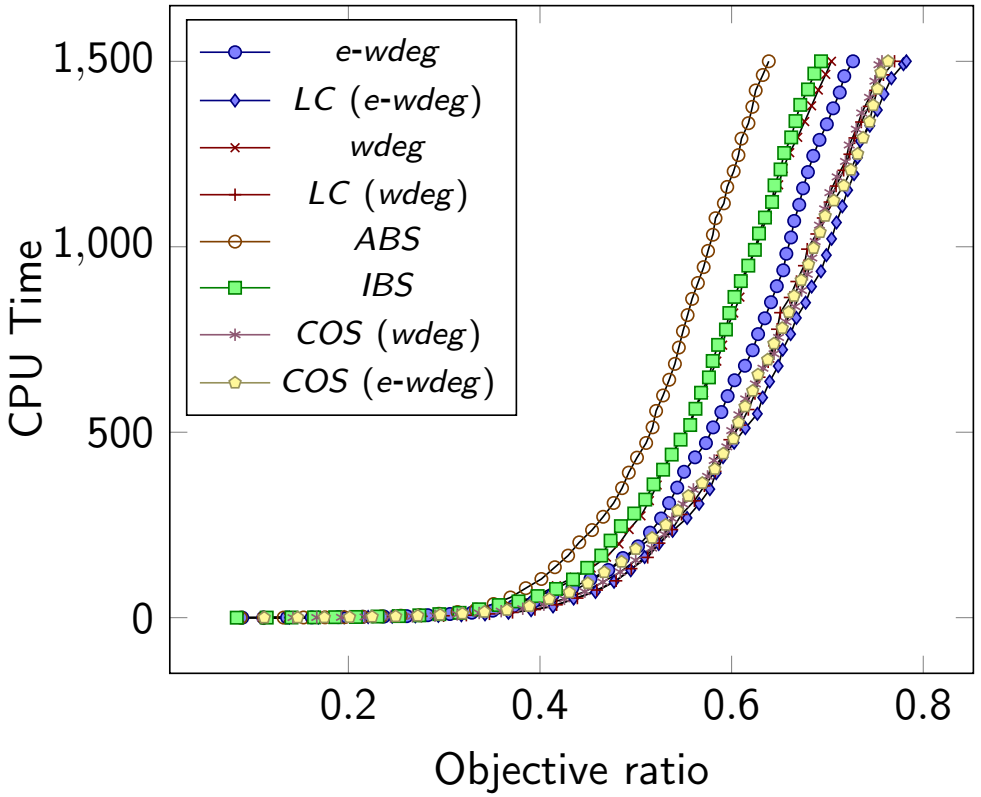

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Future research

- Design better explanations
- Explain more constraints
- How to choose between different explanations?