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Finding Robust Solutions to Stable Marriage

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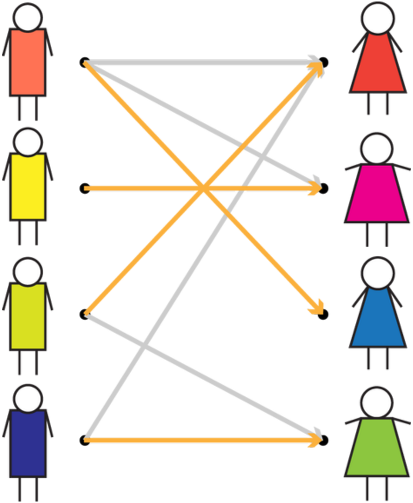
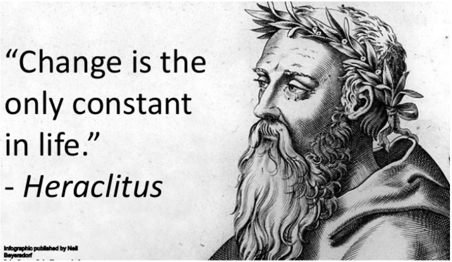
Context





“Change is the only constant in life.”
- *Heraclitus*
Infographic published by Neil Beyersdorf

Background

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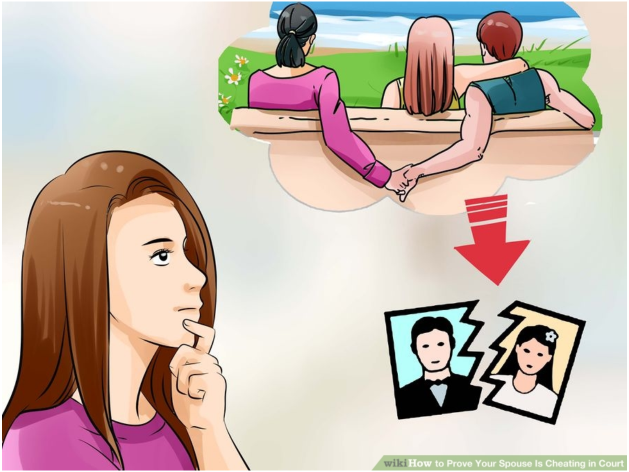
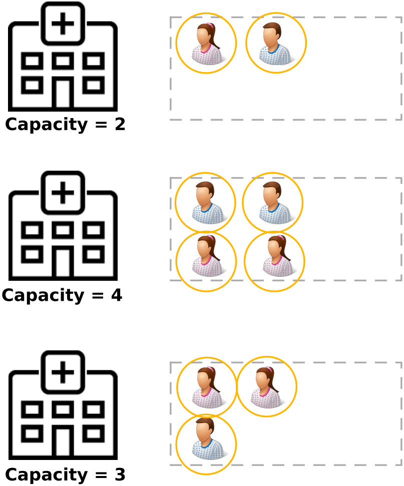
- A pair $\langle i, j \rangle$ is said to be *blocking* a matching M if i prefers j to $M(i)$ and j prefers i to $M(j)$.

Stability



Motivation



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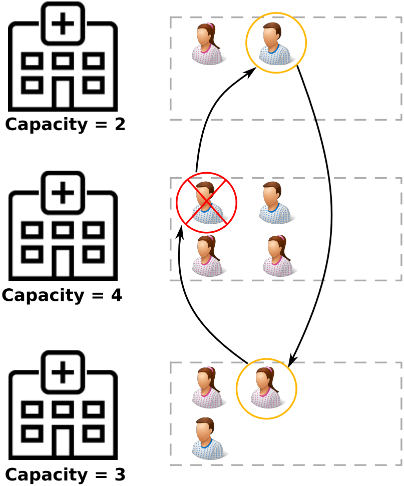


Overview of the contributions

***(a,b)*-supermatch ([Ginsberg et al., 1998, Hebrard, 2007])**
An (a, b)-supermatch is a stable matching in which if a pairs break up it is possible to find another stable matching by changing the partners of those a pairs and at most b other pairs

Overview of the contributions

(a,b)-supermatch ([Ginsberg et al., 1998, Hebrard, 2007])
An (a, b)-supermatch is a stable matching in which if a pairs break up it is possible to find another stable matching by changing the partners of those a pairs and at most b other pairs

Contributions (regarding the $(1,b)$ case)

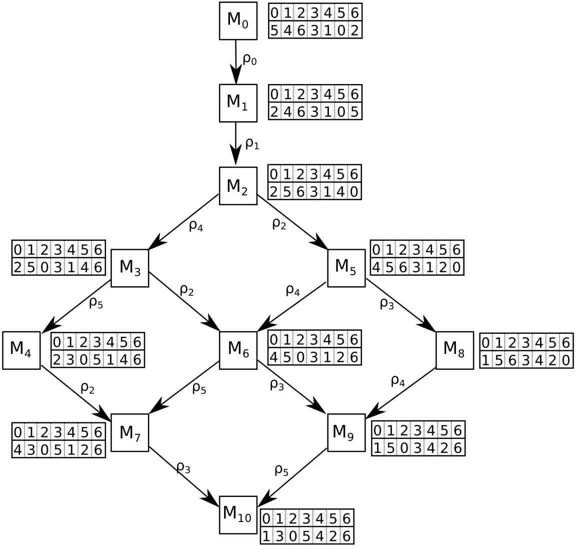
- Verification in polynomial time
- Three models to find the most robust solution
- Experimental study on random instances

Example

m_0	0 6 5 2 4 1 3
m_1	6 1 4 5 0 2 3
m_2	6 0 3 1 5 4 2
m_3	3 2 0 1 4 6 5
m_4	1 2 0 3 4 5 6
m_5	6 1 0 3 5 4 2
m_6	2 5 0 6 4 3 1

w_0	2 1 6 4 5 3 0
w_1	0 4 3 5 2 6 1
w_2	2 5 0 4 3 1 6
w_3	6 1 2 3 4 0 5
w_4	4 6 0 5 3 1 2
w_5	3 1 2 6 5 4 0
w_6	4 6 2 1 3 0 5

Lattice of Stable Matchings



Rotation

- $M_1 : \langle 0, 2 \rangle, \langle 1, 4 \rangle, \langle 2, 6 \rangle, \langle 3, 3 \rangle, \langle 4, 1 \rangle, \langle 5, 0 \rangle, \langle 6, 5 \rangle$
- $M_2 : \langle 0, 2 \rangle, \langle 1, 5 \rangle, \langle 2, 6 \rangle, \langle 3, 3 \rangle, \langle 4, 1 \rangle, \langle 5, 4 \rangle, \langle 6, 0 \rangle$

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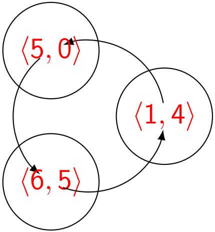

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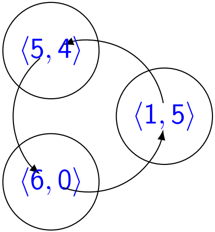

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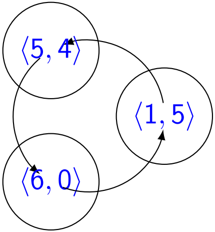
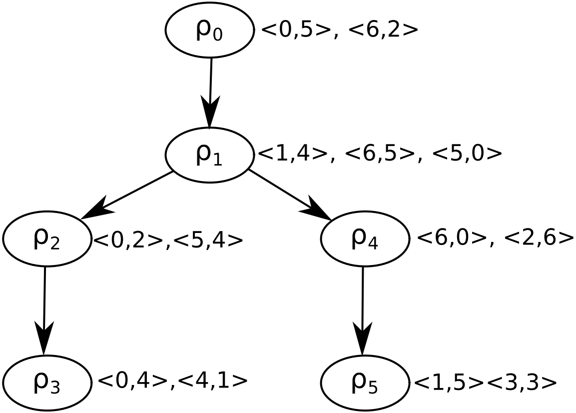


- The sequence $\rho_1 = [\langle 1,4\rangle, \langle 5,0\rangle, \langle 6,5\rangle]$ is called a rotation
- We say that $\langle 1,4\rangle$ is eliminated by ρ_1 and $\langle 1,5\rangle$ is produced by ρ_1

Graph Poset


ρ_0
<0,5>, <6,2>
ρ_1
<1,4>, <6,5>, <5,0>
ρ_2
<0,2>,<5,4>
ρ_4
<6,0>, <2,6>
ρ_3
<0,4>,<4,1>
ρ_5
<1,5><3,3>

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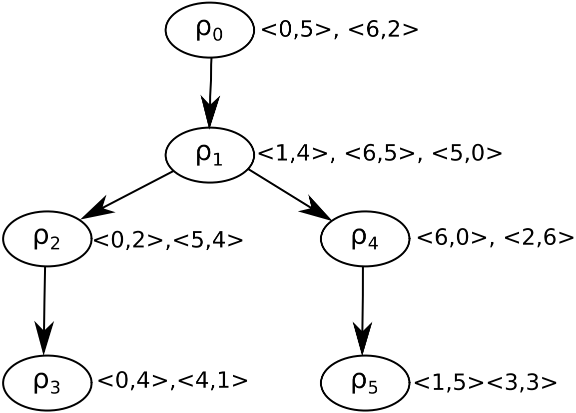
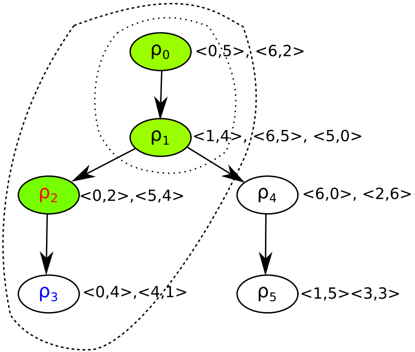
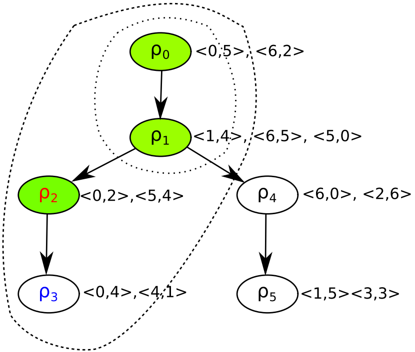


Closed Subset



Closed Subset



Theorem [Gusfield and Irving, 1989]

There is a one-to-one mapping between closed subsets and stable matchings

Verification

Verification

Verification Problem

Given a stable matching M and an integer b, is M a $(1,b)$-supermatch?

Verification

Verification

- M: a stable matching
- b: is an integer

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- b: is an integer
- S: closed subset of M
- $\langle m, w \rangle$: couple to break-up
- ρ_p: rotation that produces $\langle m, w \rangle$
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Verification

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- ρ_p: rotation that produces $\langle m, w \rangle$
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- S_{UP}: The largest closed subset $\subset S$ that does not include ρ_p
- S_{DOWN}: The smallest closed subset $\supset S$ that includes ρ_e

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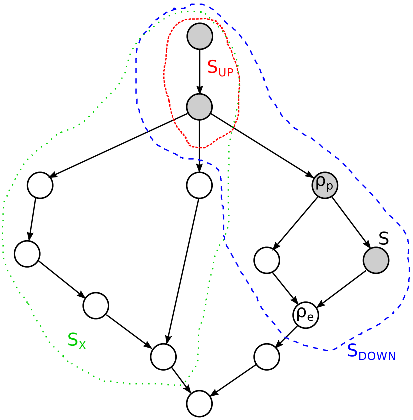


Robust Solutions

Problem

Given a SM instance, find the most robust stable matching. That is, find a $(1,b)$-supermatch such that b is minimum

Genetic Algorithm

- Random population based on random closed subsets
- The evaluation of a solution is based on the verification procedure
- Crossover: Given S_1 and S_2, pick at random $\rho_1 \in S_1$, then add ρ_1 and all its predecessors to S_2
- Mutation: Given S and a random rotation ρ, if $\rho \notin S$, then add ρ and all its predecessors to S. Otherwise, remove ρ and all its successors to S

Local Search: Key Idea

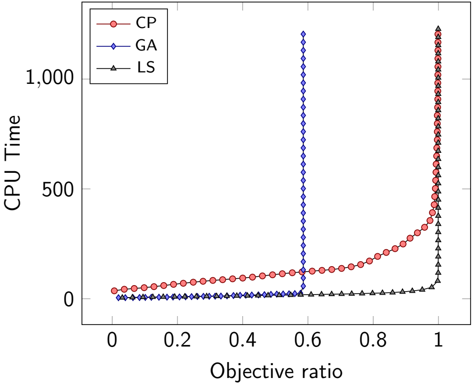
- Random solutions based on random closed subsets
- The evaluation of a solution is based on the verification procedure
- The neighbourhood of a solution S is defined by adding/removing one rotation to S

A CP model: Key Idea

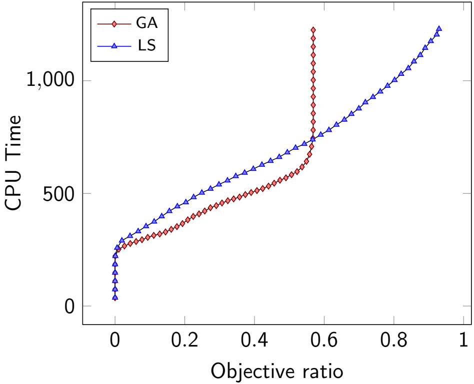
- Model stable matching using closed subsets
- Rotation variables to represent the sets S_{UP} and S_{Down} for every men
- Count for each men the repair value (b) based on the verification procedure

Experimental Study

Experimental Study



Experimental Study: Large Instances



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Future Research

- Algorithmic Complexity?
- Improve the CP Model?
- The general case of (a,b)-supermatch ?
- Other stable matching Problems?

*Picture taken from The New York Times

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